

Whitespaces after the USA’s TV incentive auction: a spectrum reallocation case study

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Abstract—Spectrum has traditionally been allocated for single uses and by now most of the “prime” spectrum has well-entrenched incumbent users. When a new service needs spectrum, there are two qualitatively distinct ways of making bandwidth available for it. A swath of incumbent users can be removed from a band, with the cleared band being reallocated for the new service. Alternatively, the new users can be allowed to utilize the interstitial spectrum holes (i.e. whitespaces) between incumbent users, with the requirement to protect the incumbents’ QoS. But these can also be used in combination by partially clearing a band and opening up the rest for whitespace-style sharing. In this case, the ability of regulators to “repack” incumbents, e.g. alter their operating channels, can reduce the need to evict them. An open question has been how whitespaces and partial spectrum clearing interact with each other and the ability to repack incumbents. **Do efficient repacks completely eliminate whitespaces?**

The USA FCC’s upcoming incentive auction in the TV bands is the first large-scale attempt to repack a major band of spectrum in order to clear spectrum for LTE. This auction is meant to navigate the tradeoff between incumbent TV services and LTE networks. In preparation, the FCC has made a large and complex data set of repacking constraints available for the first time. We have repurposed this data and built our own repacking engine in order to study a more general version of the tradeoff between whitespaces and cleared spectrum.

We conclude that (1) repacking enables clearing of significantly more spectrum than just removing incumbents; (2) the total amount of spectrum available for new uses is relatively insensitive to how incumbents are removed; (3) efficient repackings basically trade whitespace spectrum for cleared spectrum; (4) even the most efficient repackings leave plenty of whitespace — an amount that can be comparable with the amount of cleared spectrum.

I. INTRODUCTION

TV spectrum has recently become a very popular topic due to its proximity to mobile spectrum as well as the TV whitespaces, which give access to spectrum necessary for economic development. There are many interesting aspects to the field of cognitive radio and whitespaces, such as coexistence techniques, network planning, system architecture, and security and robustness, whose unique challenges have been studied to varying degrees. However, few studies address a very simple question: when is it better to completely reallocate a band vs. to share it?

In fact, there are several different options for making “new” spectrum, as shown in Figure 1:

- 1) *Completely reallocate the band as a single-use band.* Until recently this was the standard way of reallocating spectrum. Complete clearing is especially useful for applications which cannot or will not share spectrum.

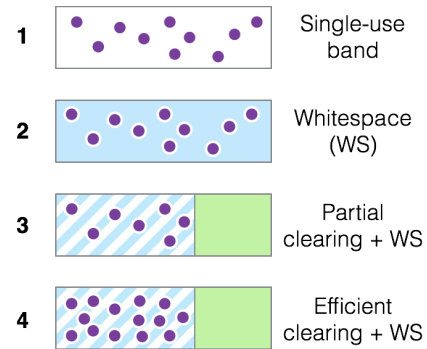


Fig. 1. An illustration of the various options for spectrum repurposing. Incumbents are shown as purple dots while whitespaces are blue and cleared spectrum is green. White represents unused spectrum (in the case of whitespaces, this is a buffer which is necessary to maintain the incumbent’s quality of service). The white and blue hashed pattern represents spectrum that could but need not support whitespace rules.

- 2) *Declare the entire band potential whitespace while preserving the quality-of-service of the incumbents via sharing rules.* This is becoming the de facto way of “generating” new spectrum, especially after the publication of the PCAST report¹ [1]. Whitespace regulations naturally have to navigate a tradeoff between quality-of-service for the incumbent vs. the secondary users. This has been explored in [2]–[4].
- 3) *Partial clearing of the band.* Pristine spectrum is created while a portion of the incumbents remain. The uncleared spectrum may be designated as either single-use spectrum or as whitespace with the incumbents as the primary users. Partial clearing is preferable when it is not possible or desirable to remove all of the incumbents.
- 4) *Efficient partial clearing of the band.* The spirit and use cases are very similar to scenario 3 except that this option maximizes the number of incumbents that remain after a partial clearing. Rather than remove the incumbents which were in the now-cleared spectrum, these incumbents are efficiently packed into the remaining (uncleared) spectrum whenever possible. This approach essentially sacrifices would-be whitespace in order to

¹This report, submitted as a recommendation to the President of the United States by the President’s Council of Advisors on Science and Technology in 2012, emphasized the need to find at least 1,000 MHz of spectrum as soon as possible and highlighted spectrum sharing as the best way to accomplish this goal.

“house” an incumbent². Thus more incumbents remain in service at the expense of secondary users.

For years, only the first three scenarios were easy to analyze. However, the FCC’s upcoming incentive auctions provide an excellent chance to study the fourth option in the context of the United States. Briefly, the incentive auctions give television broadcasters a chance to bid one of two choices: (1) relinquish their spectrum usage rights or (2) be “repacked” (i.e. moved to another channel) within the TV bands³. As overseer of the entire auction, the FCC will subsequently decide which stations will be “repacked” vs. removed, in the process creating a situation akin to scenario 4. The cleared spectrum will be auctioned off in a manner which encourages prospective LTE-system builders (e.g. AT&T, Verizon, Sprint, and T-Mobile) to buy it. In the interest of space and focus, we have simplified the incredibly complex incentive auction. We encourage interested readers to read our appendices [6].

“Repacking” incumbents is not a trivial task (as proof: the FCC is currently being sued by the National Association of Broadcasters over its repacking methodology [7]). However, the FCC has made their repacking process and constraints public [8] which gives us the opportunity to conduct hypothetical spectrum repacks of our own, independent of the auction’s actual course. Thus in this paper we use the incentive auction data to explore the fourth spectrum-scrounging scenario.

We first look at how many incumbents would need to be removed in order to meet a variety of spectrum clearing targets and show that the ability to efficiently repack drastically reduces the need to remove incumbent users. We show that repacking also concentrates the removal of incumbents to the geographic areas where it is strictly necessary, reducing unnecessary loss of incumbent services. Although efficient repackings aim to pack the spectrum as tightly as possible, we find that interstitial spectrum holes remain even in the places which already had few whitespaces.

Finally, we explore the true tradeoff between incumbent services (e.g. TV) and spectrum for new services. We find that for the same sacrifice of incumbent service, the total spectrum available to an opportunistic device that uses both TV whitespaces and cleared spectrum is surprisingly insensitive to the clearing method. The effect of repacking efficiency is in modulating the tradeoff between whitespaces and cleared spectrum, rather than between incumbent and new services. In particular, more efficient repackings reduce whitespaces but create an equivalent amount of cleared spectrum.

A. Prior work

The idea of incentive auctions can be traced back to the proposal in 2002 by Kwerel and Williams of the FCC, which pressed for a rapid transition to a market-based allocation of spectrum and specifically called for a large-scale two-sided

²Note that this is not an option in smaller bands where the incumbent has a fixed bandwidth equal to the width of the band. However, there are many channelized bands which would be good candidates for this kind of solution.

³Broadcasters are actually given several choices [5, ¶365] but in this paper we simplify to the most important choice for brevity and clarity.

auction for repurposing spectrum from incumbents who are willing to relinquish their rights [9]. The National Broadband Plan of 2010 [10] proposed the use of such a two-sided “incentive auction” for repurposing spectrum from broadcast television services to mobile broadband services (LTE). In February 2012, Congress authorized the FCC to conduct these auctions through what came to be known as the Spectrum Act [11]. Soon after, the FCC announced the preliminary plan for this auction in their October 2012 Notice of Proposed Rulemaking (NPRM) [12]. A report and order released by the FCC in June 2014 laid out the structure of the auction process [5] and a follow-on Public Notice released in December 2014 [13] provided additional details. Since its inception, the economic value, potential impact, and complexity of this unique auction has generated a lot of interest in both the business and academic communities. At least three major mobile broadband providers, AT&T, Verizon Wireless and T-Mobile, have commissioned teams of researchers to perform speculative analyses of the auction [14]–[17].

Arguably the most novel and challenging feature of the incentive auction process is its intricate entanglement with the repacking problem, which concerns allocating a set of stations to a set of channels given constraints motivated by the physical nature of the problem. This problem can be naturally posed as a large scale Boolean Satisfiability problem (see [18]) with tens of thousands of variables and hundreds of thousands of clauses in a typical instance.

The constraints themselves have been released on FCC’s LEARN website [19], a website intended to help the public understand how the incentive auctions work. The data is in the form of two files. The first file lists, for each of the 2,173 repack-eligible TV stations, the list of channels to which it may be assigned⁴. The second file contains 291,739 entries, each of which details the co- or adjacent-channel interference constraints between TV stations (e.g. “station A may not operate cochannel to station B on channel C”).

A study of this repacking problem focusing on the computational difficulties and the performance of SAT solvers can be found in [18]. Further, the FCC has conducted a public workshop to help disseminate information about this complex problem, a webcast of which can be found online [20]. Recently, using the data released by the FCC, an analysis of the feasible repackings corresponding to a variety of contingent spectrum clearing targets was done in [14] (commissioned by AT&T). We use some of the techniques developed there as the starting point of our analysis.

B. Brief overview of our methods

To generate the various sets of data used in this paper, we rely on data from the FCC’s LEARN website [19], as described above. We then use PycoSAT, a Python wrapper for the well-known SAT solver library PicoSAT [21], in order to synthesize the constraints and output a feasible repacking.

⁴External factors, such as harmonization with Mexico and Canada, sometimes prevent the assignment of a particular station to a particular channel.

Two other studies on repackings have used PicoSAT [14], [16] and it was also featured in an FCC workshop on the topic of repacking in the incentive auctions [20].

Finally, we build on this data via our Whitespace Evaluation Software (WEST), an open-source toolbox for computing the amount of available whitespace [22]. Complete methodological details are in the appendices of this paper [6].

Note that because it is a combinatorial problem, there are many candidate station assignments that achieve the same goal (e.g. clear N channels by removing exactly M stations). For each possible scenario, we generate 100 candidate assignments which are later presented as aggregate statistics. Typically, we will use the median value (as taken over all assignments) as this is a standard and robust metric.

II. HOW MANY INCUMBENTS MUST BE REMOVED TO FREE SPECTRUM?

There’s no such thing as a free lunch. However, there are good and bad ways of removing incumbents in order to clear new spectrum. A naive way is to remove precisely the incumbents which happen to be in the channels to be cleared, i.e. scenario 3 in Figure 1. However, this immediately leads to a few problems:

- 1) More incumbents will be cleared than necessary – compare the two lines of Figure 2. The difference between these lines represents the number of stations that can be “removed” via repacking rather than taken off the air.
- 2) In some cases (e.g. with TV) it can be difficult to assess the value of the incumbent. So although we could consider invoking something akin to eminent domain, it’s unclear what the fair market price would be. This means that any offered price would likely be challenged, delaying the reallocation of spectrum and creating uncertainty for all parties.
- 3) If only the incumbents which happen to be in the channels to be cleared participate in the market, the lack of competition could potentially lead to obvious problems such as holdouts (stations demanding unreasonable sums of money because they have a good bargaining position).

For these reasons it is important to any market-based clearing process that we have a means of substituting one station for another in order to foster competition. In the incentive auctions, this substitutability is facilitated by the ability to repack TV stations.

The blue line in Figure 2 shows the results of our computations for determining the minimum number of stations that *must* be removed in order to meet different spectrum clearing targets. The efficient-clearing numbers are substantially lower than the naive approach of removing all stations which happen to be in the desired band.

We perform these computations by building on the techniques in [14], where a similar computation was done with additional constraints that are motivated by those of the actual incentive auction (e.g. a station may only be reallocated to a channel near its original channel). We found that about 10% fewer stations need to be removed to meet the same

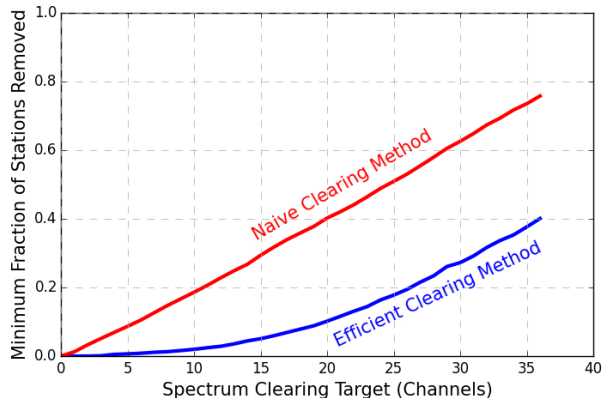


Fig. 2. Minimum number of incumbents (TV stations) that need to be removed to meet each spectrum clearing target. In this figure we compare a naive clearing method (scenario 3 in Figure 1) and an efficient clearing method (scenario 4). Efficient clearing is accomplished via “repacking” (moving in frequency but not space) existing TV stations rather than simply removing them.

clearing targets as compared to those reported in [14]. This was partially a result of exploiting certain properties of the SAT solver to improve its performance. The numbers could indeed be even lower; further improving the performance of SAT solvers on these problems is a topic for future work.

III. TELEVISION AVAILABILITY AFTER REPACKING

Another way of looking at the impact of the results in Figure 2 is to examine which places lose access to TV. Figure 3 shows which places lose at least one TV channel under the naive clearing method (orange), the efficient clearing method (green), or both (blue) when 14 TV channels are to be cleared.

Our first observation is that there are almost no places where TV availability is impacted in the efficient clearing method but not the naive method. We further see that with the efficient clearing method, only stations in the most populous markets are affected. In contrast, the naive clearing method impacts a very large fraction of the population in addition to having a much greater impact on rural areas. Thus the ability to repack stations as opposed to simply removing them helps us confine the impact to only those places which are unavoidably impacted. We have provided more detailed maps showing how many TV channels were lost in each market⁵ in the online appendices of this paper [6].

Beyond answering the question of how much TV coverage will be lost, this figure is important because it gives insight into how the repacking will work. In particular, we see that the areas which currently have a lot of TV stations (e.g. New York City, Los Angeles) have so many that some must be removed rather than repacked. However, most of the country is not brimming with TV stations (as evidenced by the current

⁵In the efficient clearing method, San Francisco, Los Angeles, and the most populous portions of the east coast lose more than four channels while other areas do not. With the naive clearing method, a variety of regions (including e.g. Utah) lose 6 or more TV channels.

amount of whitespace in these regions) and so no stations would need to be removed to meet most clearing targets.

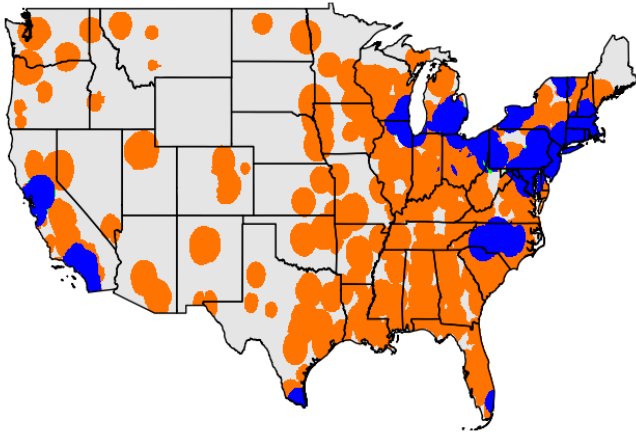


Fig. 3. Map showing which locations in the United States may lose TV coverage after the incentive auctions if 14 TV channels are repurposed using the naive clearing method (orange), the efficient clearing method (green), or in both cases (blue). Gray denotes areas whose TV coverage is not affected.

IV. WHITESPACES REMAIN AFTER REPACKING

The key question regarding the incentive auction within the cognitive radio community is “will whitespaces still exist after the incentive auctions?” The answer to this is quite simply: yes. In this section we will look at the expected *minimum* amount of whitespace that will remain for each spectrum clearing target. As before, we will consider the case where the minimum number of incumbents are removed. If more incumbents are removed for the same clearing target, *more* whitespace will be made available⁶.

There are several ways to measure the amount of whitespace that will remain after the auction, such as:

- 1) Raw amount of whitespace after the auction
- 2) Change in whitespace due to the auction

Figure 4 shows the predicted number of whitespace channels available to fixed whitespace devices if the incentive auction clears 7 TV channels. We see that the major population centers are indeed feeling a bit of a spectrum crunch (as they always have) but that at least 30 percent of the population has more than 15 whitespace channels.

To get a better sense of how much whitespace is likely to remain after the auctions, we turn to Figure 5. Two sets of CCDFs are shown, one for fixed devices and the other for portable devices⁷. CCDFs for the current assignment of TV

⁶Each repacked incumbent will be protected and hence will diminish the amount of whitespace available. Fewer incumbents means fewer restrictions on whitespaces.

⁷The main distinction between fixed and portable devices is that fixed devices may not operate in a channel adjacent to a TV station whereas portable devices may do so (at a lower power). In this paper we do not consider the standard operating-channel restrictions applied to portable devices because we expect those to change after the incentive auction [23]. It is also worth noting that fixed devices may be able to operate under portable-like rules (adjacent-channel but with lower power) in the future.

stations are shown in black for comparison and the other lines represent the expected minimum amount of whitespace that will remain with a variety of spectrum clearing targets. The median US citizen will have approximately 4 whitespace channels (24 MHz) available for fixed devices (and 14 channels—84 MHz—for portable devices) in an auction which clears 21 TV channels (the red line), one of the more optimistic auction outcomes in the FCC’s eyes.

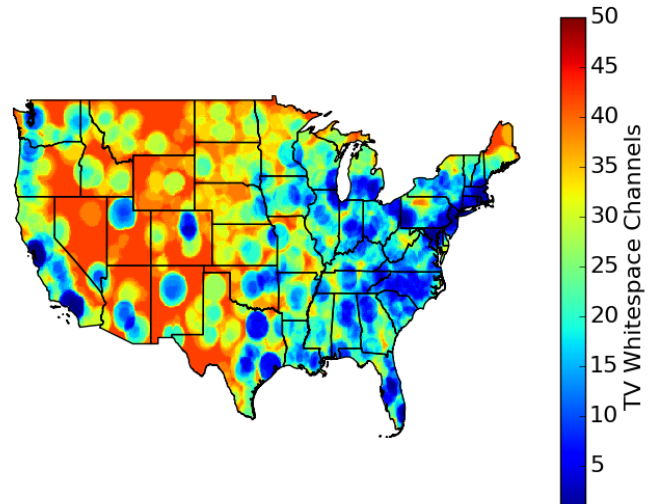


Fig. 4. Number of expected whitespace channels for fixed devices if 7 channels are reallocated in the incentive auctions.

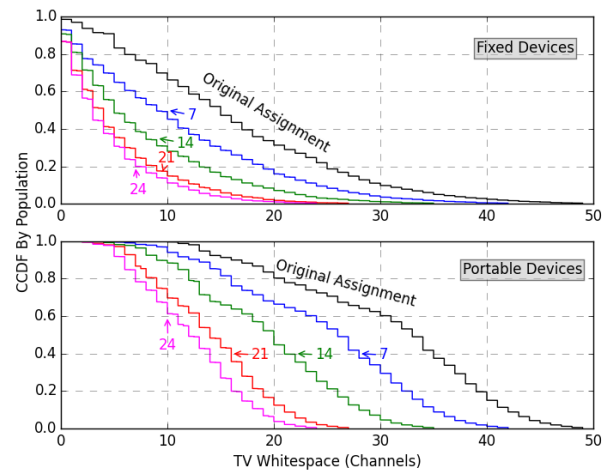


Fig. 5. CCDF weighted by population showing the minimum amount of TV whitespace which is expected to remain after the incentive auction. Note that cleared spectrum is *not* included in these numbers.

Next, we directly compare the expected amount of whitespace after the auction to the current amount of whitespace. This is shown via 2-D histograms in Figure 7 for portable devices and Figure 8 for fixed devices. Red indicates that many people fell into that particular category whereas blue indicates an event affecting fewer people. The two diagonal

lines correspond to (1) having lost no whitespace (the top line) and (2) having lost an amount of whitespace which is equal to the number of channels cleared in the auction. The horizontal line indicates the maximum possible number of whitespace channels (channels above this line have been cleared and are not counted as whitespace). We describe the generation of these two-dimensional histograms in greater detail in the online appendices [6].

People who lose no whitespace in the auction are often in places where a TV station was removed in order to meet the clearing target, and so the channel that was cleared was never available as whitespace in the first place. In essence, TV channels were taken away, not whitespace channels. We can see from Figure 3 that this is predominantly in urban areas. People near the second diagonal line had enough free channels that all of their TV stations could be repacked, thus channels that were previously whitespace are the ones that have been cleared. Although this happens mostly in areas of low population density, the sheer quantity of them means that they represent almost 50% of the US population for the situation in Figure 3.

We note a few important features of these plots:

- 1) As is well-known, fixed devices in general have less available whitespace than portable devices (this is due to differences in the regulations for these devices).
- 2) When few channels are reallocated, the mass is mostly clustered around the lower diagonal line. The distribution of post-auction whitespaces becomes more variable if more channels are cleared.
- 3) In the case of fixed whitespaces, most people have few channels to begin with but tend to keep them.
- 4) Portable devices are more likely to experience a reduction in whitespaces. Essentially they “had more to lose.”

Finally, Figure 6 compares the expected whitespace after a repack with N channels cleared (the y axis) to the amount of whitespace resulting from a naive reallocation which also results in N channels cleared (i.e. scenarios 3 and 4 in Figure 1). We see that the majority of the mass is falling below the $y = x$ line, indicating that whitespaces are more plentiful with the naive reallocation. The extra whitespace in the naive reallocation indicates an inefficient use of spectrum for the primary as compared to the appropriately-named efficient allocation.

V. WHITESPACES VS. REALLOCATION

As mentioned in the introduction and shown in Figure 1, there are several basic ways to create new spectrum opportunities. We explore these options in terms of the tradeoff between delivered services for the incumbent vs. a new spectrum-hungry device. We consider three types of devices: (1) those that want their own dedicated bands (e.g. LTE devices); (2) those that can operate only in the TV whitespaces; and (3) devices which are willing to harvest any spectrum possible.

Figures 9 and 10 both show this tradeoff in terms of the number of over-the-air TV channels the median US citizen could watch versus the amount of spectrum that the median

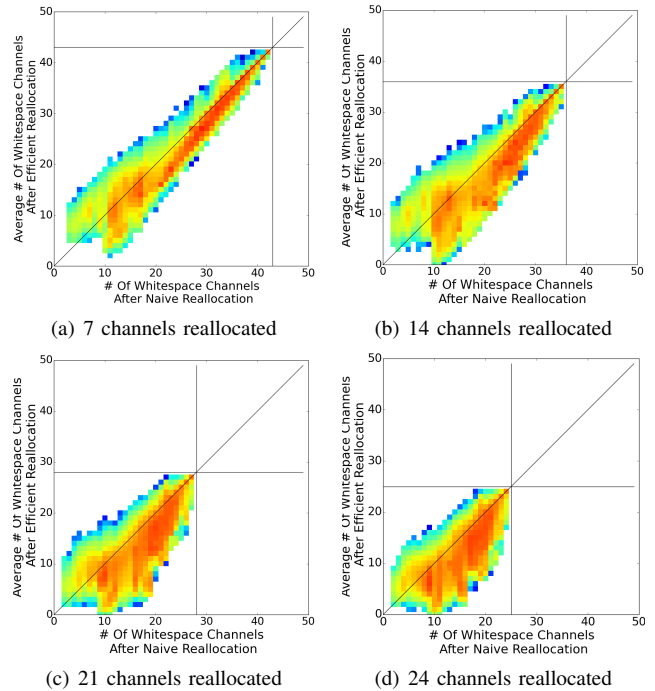


Fig. 6. Direct comparison via 2-D histogram of whitespaces in scenarios 3 and 4 of Figure 1. Pixels are colored based on the number of people in the bin with red indicating the most people and blue the fewest.

citizen could access for different spectrum clearing targets. (We describe the process of construction of these figures in greater detail in the appendices of this paper [6].) Each figure has two types of lines which parallel those in Figure 2:

- 1) Dashed: naive removal method (stations are removed from highest channels first; no assignments in uncleared spectrum are modified). This is scenario 3 in Figure 1.
- 2) Solid: minimal removal + optimized repacking method. This is scenario 4 in Figure 1.

In particular, Figure 9 shows the amount of completely cleared spectrum (in purple) as well as the amount of spectrum that could be obtained by an opportunistic portable device which can operate in either the TV whitespaces or the cleared spectrum⁸ (in red).

When no TV viewership can be sacrificed (i.e. the top points on all three lines), being able to use whitespaces significantly increases the amount of available spectrum. However, sacrificing even one watchable TV channel (for the median US citizen) with efficient clearing gets us almost as much cleared spectrum as fixed whitespace. Portable whitespace will always outperform fixed whitespace because of the nature of the protections afforded to the incumbents which depend on the device type.

⁸The spectrum cleared in the incentive auctions will be available for use by whitespace devices subject to rules that protect the spectrum purchaser’s rights as a primary user of the spectrum [5, ¶678]. Because it is difficult to predict what the distribution of new primary users will be or even what their protection criteria will be, we treat cleared spectrum as 100% available (but not whitespace) for the purposes of this paper.

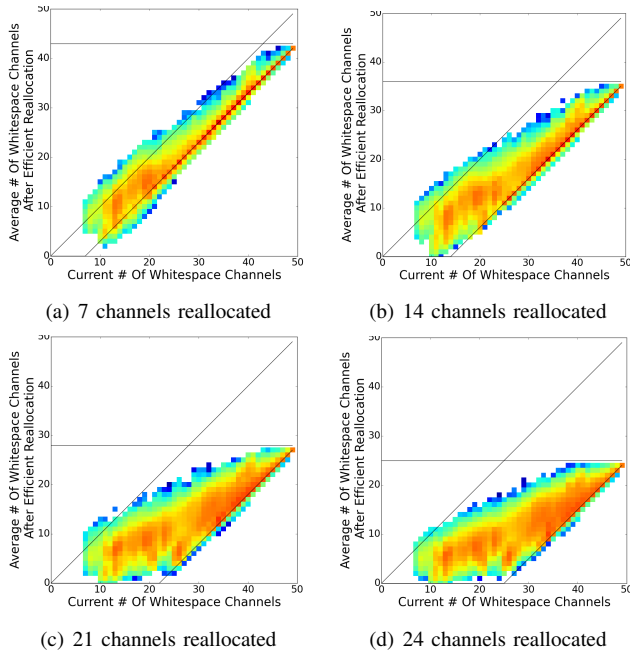


Fig. 7. Direct comparison via 2-D histogram of whitespaces available to portable devices before and after the auction. Pixels are colored based on the number of people in the bin with red indicating the most people and blue the fewest.

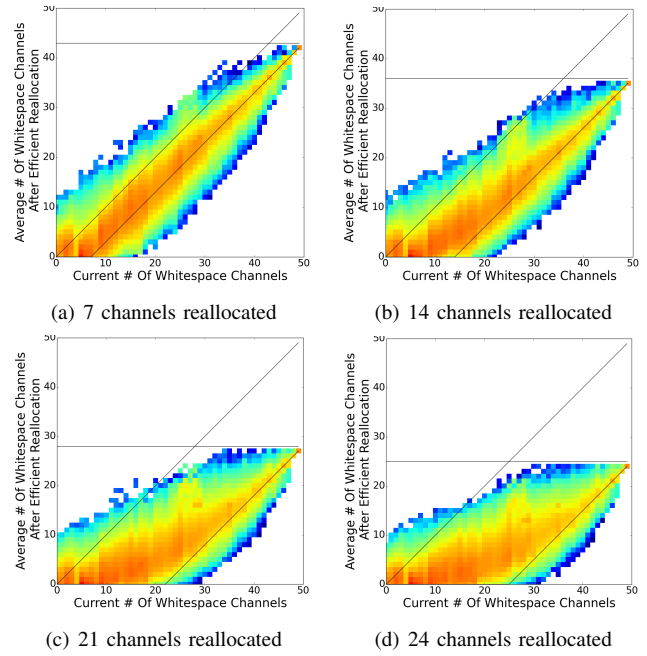


Fig. 8. Direct comparison via 2-D histogram of whitespaces available to fixed devices before and after the auction. Pixels are colored based on the number of people in the bin with red indicating the most people and blue the fewest.

The most interesting curves are the right-most ones (in red) in Figure 9. These curves show that the total of cleared spectrum and portable whitespaces is essentially the same under the naive clearing method and the efficient clearing method. Moreover, that final curve has a clear slope of -6 : to gain 6 MHz (one channel) of available spectrum, one watchable TV channel must be sacrificed. This shows that from the perspective of the TV-vs.-new-device tradeoff, the repacking method affects the *balance* between cleared spectrum and whitespace spectrum, *not* the total amount of spectrum available to an opportunistic whitespace-like device. In terms of total-spectrum, it is essentially a zero-sum game at the margin.

To understand this, it is useful to reorder the curves to focus on the whitespaces, as is shown in Figure 10. The red line (total spectrum) is repeated for illustrative purposes. Here, first notice that TV whitespaces and watchable TV channels shrink *together*. While this is not the normal look of the TV-vs.-whitespaces curve, the effect is explained by the fact that cleared channels take away from both TV and whitespaces. The behavior is intuitive because only the interstices within *TV spectrum* are deemed whitespaces and as the total size of the TV band shrinks, it is natural to expect the whitespaces to shrink along with it. Next, notice that for the same amount of watchable TV, the naive clearing method gives rise to far more fixed and portable whitespace than the efficient clearing method. This is because the efficient clearing method is in effect filling in the whitespaces with repacked stations that are being relocated down from the cleared spectrum instead of being evicted.

Whitespace can take advantage of assignment inefficiencies in a way that reallocation simply cannot, so in any practical scenario there will always be a significant whitespace gain. For example, even in the 24-channels-cleared plan that the FCC is considering to reallocate 144MHz of spectrum to LTE, we anticipate there will still be 75MHz of portable whitespace left in the remaining TV bands for the median US citizen.

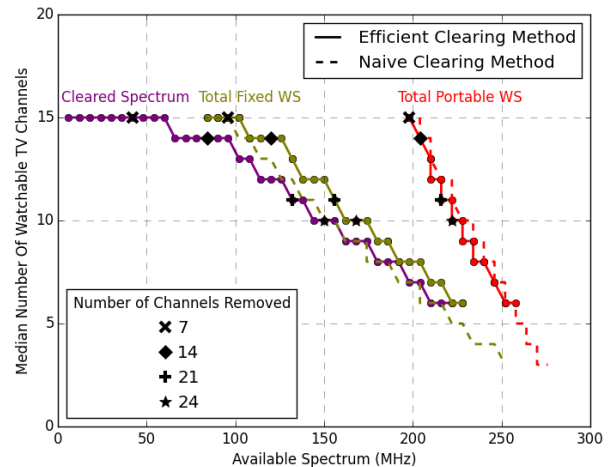


Fig. 9. Tradeoff between over-the-air TV service and available spectrum. This takes a cleared-spectrum centric view with the first curve being cleared spectrum and the subsequent curves adding in the amount of fixed and portable whitespace, respectively. Note that the “Total Portable WS” line in this figure is the same as the “All Mobile Spectrum” line in Figure 10.

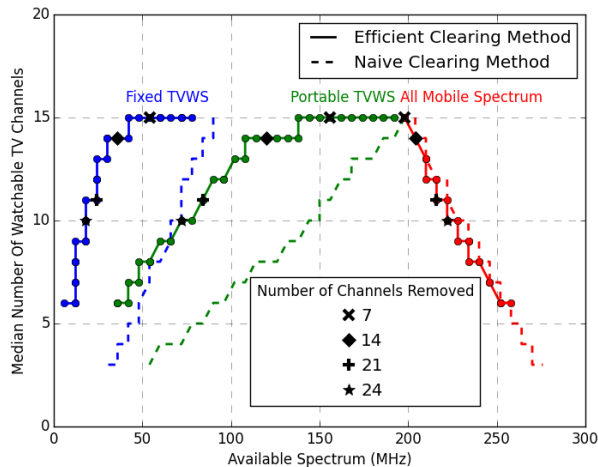


Fig. 10. A whitespace-centric view of the tradeoff between over-the-air TV service and available spectrum. Note that the “All Mobile Spectrum” line in this figure is the same as the “Total Portable WS” line in Figure 9.

VI. CONCLUSIONS

This paper has opportunistically used the upcoming TV spectrum incentive auction in the USA as a way to examine the tradeoff between clearing spectrum and enabling the use of whitespaces.

We saw that, with efficient clearing, over-the-air TV availability will be affected primarily in major metropolitan areas where it is necessary to evict TV stations in order to achieve most spectrum clearing targets.

Overall we conclude that:

- 1) Repacking enables clearing significantly more spectrum than does just removing incumbents.
- 2) The total amount of spectrum available for new uses is relatively insensitive to how incumbents are removed.
- 3) Efficient repackings basically trade whitespace spectrum for cleared spectrum.
- 4) Even the most efficient repackings leave plenty of whitespace spectrum — an amount that can be comparable with the amount of cleared spectrum.

This supports the idea that spectrum sharing between heterogeneous uses is essential for full utilization of spectrum [24].

VII. ACKNOWLEDGEMENTS

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